On the high-energy Elastic Scattering of hadrons at large t

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Abstract

The main contribution to hard elastic scattering comes from components of wave functions of colliding hadrons that contain minimum number of partons. We discuss this mechanism in regge and parton approaches and estimate the probabilities that colliding hadrons are in such bare states. The behavior of cross-sections in this regime at various energies can give nontrivial information on high energy dynamics.

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1. Introduction.

The elastic scattering at high energies is reasonably well described by the regge approach. At small t it has the diffractive nature and is connected with the pomeron exchange. For the supercritical pomeron $I\!P$ (such structure of $I\!P$ is suggested by experiments and by the QCD calculations) the mean number of $I\!P$ exchanges increases with energy. And gradually, at asymptotic energies, the power grow of amplitudes $A \sim \exp(\Delta y)$ changes, as usually expected, to the Froissart type behavior $A \sim y^2$, where $y = \log s/m^2$ is the full rapidity. The width of diffraction peak $\delta t \sim 1/R^2(y)$ at not very high y is defined by the regge radii of colliding objects $R^2(y) \simeq 2R_0^2 + \alpha' y$. The diffractive contributions decrease fast with $k_\perp^2 = -t$ - approximately as $\exp[-k_\perp^2 R^2(y)]$.

The character of interaction changes at subsequent grow of $k_\perp$ - from diffractive to the direct parton exchange. For the experimentally investigated energies this transition takes place at $|t| \simeq 2 \div 5 \ GeV^2$. In regge approach this transition zone is smeared, because, at first, with grow of $k_\perp$, the mean number of $I\!P$ exchanges also grows $\sim k_\perp$, and the elastic amplitude can be approximately represented by the Orear type expression $\sim \exp(-k_\perp f(y))$. In this case the large transferred momentum $k_\perp$ is distributed approximately uniformly among the exchanged pomerons, and this is connected with the approximate linearity of regge trajectories at small t.

For even larger $k_\perp$ this uniform distribution changes, and the large transferred momentum $k_\perp$ is concentrated mainly on a single $I\!P$ line. This is primarily related with the nonlinearity of the $I\!P$ trajectory and its hard satellites in QCD, which at large $k_\perp$ must approach to the fixed values as $\alpha_P(k_\perp) \simeq 1 + c/\log(k_\perp^2)$ [1]. In this case the t-behavior of amplitude is defined by the pomeron vertices, and one can expect the power behavior of full regge amplitudes $A \sim 1/k_\perp^{2\nu}$, where ν is determined by the minimal number of exchanges needed to scatter all parton components of hadron on the same angle. In this mechanism the main contribution to the scattering amplitude comes from components of wave functions of the colliding hadrons with minimal possible number of partons. The picture closely corresponds to the

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[1]Note that, as can be seen from a comparison with the experimental data, the regge trajectories look linear up to rather large $-t \simeq 2 \div 5 GeV^2$, and only after that they can move to the bare values. The details of this phenomenon are not so well investigated, and it is, probably, related to the large nonperturbative contributions, essential up to sufficiently small distances $\sim 0.1 \div 0.2 \ fm$

quark counting rules [1] and its development [2]-[3], and the corresponding behavior of cross-sections is approximately seen in experiments.

Such high $k_\perp$ scattering mechanism contains two main ingredients. One is the amplitude of hard scattering of valent (bare) constituents. It is defined by the perturbative QCD, and can be estimated even on the dimensional grounds. The other is the amplitude to find hadron in such a "bare" state, and it is defined mainly by the nonperturbative physics. What one first of all needs is the dependence of this amplitude on the energy of hadrons and the transverse resolution. This is what we consider below in regge and parton approaches.

2.   Cross-sections

Consider the contribution to the elastic scattering amplitude $A(s,t)$ from rather general reggeon diagrams of Fig.1 at high $|t| \gg 1/R^2$, but such that $|t| \ll s$. Because, as we have assumed, the $I\!\!P$ trajectories are nonlinear, and freeze at high $-t$, the $I\!\!P$ exchange amplitude $v \sim ig^2(t)s^{\alpha(t)}$ behaves at high $-t$ in power-like manner [2]. As a result in this case the main contribution to A comes from diagrams Fig.1a in which almost all transverse momenta is transferred trough a single $I\!\!P$ line. The full diagram for A can be symbolically represented as in Fig.1c with the contribution

$$A(s, t = -k_\perp^2) = \int d^2q_{1\perp}S(s, q_\perp^2)v(s, (k+q)_\perp^2) \simeq$$

$$\simeq v(s, k_\perp^2) \int d^2q_{1\perp}S(s, q_\perp^2) = v(s, k_\perp^2)\tilde{S}(s, 0) , \qquad (1)$$

where we separated integration over the $I\!\!P$ line with high momentum transfer. Here $\tilde{S}(s, b)$ is the elastic S-matrix in impact parameter plane b, and in (1) it enters at zero impact parameter, reflecting in particular that at high

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[2] The structure of pomeron in QCD can be rather complicated and $I\!\!P$ probably consists of sequence of regge poles $I\!\!P_n$ with intercepts $\simeq 1 + c/n$, and small t slope $\sim 1/n^2$, and such that their internal virtuality grows like $q_\perp^2 \sim \exp n$. At high $-t$ all these satellite $I\!\!P_n$ accumulate at their bare values like $\alpha_{nP}(k_\perp) \simeq 1 + c/(n + c_1 \log(k_\perp^2/\Lambda_c^2))$. At high momentum transfer the contribution of satellites $I\!\!P_n$ with high n in the amplitude can be even dominant - so that the essential $n \sim \log(-t)$. But here we will not take into account these details because all $I\!\!P_n$ trajectories move at high $-t$ to the same bare value, and the structure of exchange is defined by the minimal perturbative graph.
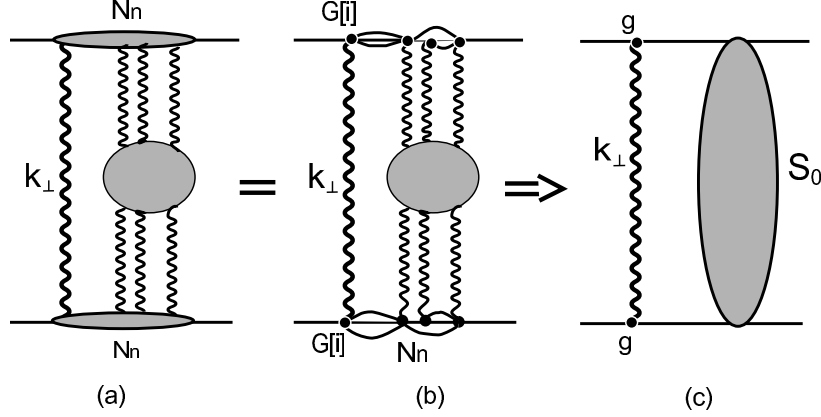
Fig.1 : **(a)** General reggeon diagram for elastic multi-$I\!\!P$ apmlitude. The $I\!\!P$ line with high $k_\perp$ is selected. **(b)** The decomposition of the multipomeron vertexes N_n into jets. **(c)** Diagram essential at large $k_\perp$.

momentum transfer the small b are essential. This finally gives

$$\frac{d\sigma}{dt} \simeq |S_0(s)|^2 \frac{d\hat{\sigma}}{dt} \quad , \qquad S_0(s) \equiv \tilde{S}(s, b=0) \quad , \qquad (2)$$

where $d\hat{\sigma}(s,t)/dt$ is the "bare" cross-section, connected with hard exchange. The quantity $\tilde{S}(s, b=0)$, entering (1), represents the contribution from the interaction in the initial and final state, and the $\tilde{S}(s, b)$ is equal to the amplitude that one of colliding particle moves trough another without interaction at given s and impact parameter b. One factor $|S_0(s)|$ in (2) can be interpreted as a probability to find the fast particles in such a "bare" state that they almost do not interact with a target. Another factor $|S_0(s)|$ reflects that scattered particles in the final state are also in such a bare state. In the lab. frame of one of colliding particles the $|S_0(s)|$ gives the probability that another particle is in the state without a soft parton cloud [3].

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[3] In logarithmic theories like QCD one should (in principal) include in a parton cloud also all partons with $q_i^2 \leq k_\perp^2$, which can be separately exited in such a hard process. But at not too high energies there are low partons with high $k_\perp$. It is seen also even at the LHC energies where the mean $k_\perp^2$ almost do not grow.

The essential point which has to be taken into account when going from diagrams Fig.1a,b to Fig1.c , is connected with the structure of multipomeron vertices N_n. In the general contribution of Fig.1b the multiparticle diffractive jet goes out from the hard $I\!\!P$ vertices $G[i]$, and then in resulting expressions (1, 4) one should sum over the states [i] of such jets. But the particles produced in such a hard diffraction would have large relative transverse momenta of order $k_\perp$. Particles lines with these high $k_\perp$ enter neighboring vertices and will lead to an additional smallness $O(1/k_\perp^2)$ of corresponding contribution to the amplitude. Alternatively, this jet can be especially aligned in order to compensate the large transferred momentum to other vertices – the corresponding small probability results in the same small factor in multiparticle jet contribution. Therefore at high $k_\perp$ in the states outgoing from the hard vertices g in Fig.1c only one particle state survives. Such an answer looks natural because in this case the minimal number of constituents pass trough the hard vertex.

In the factorized form (2) the quantity S_0^2 represents the contribution of large transverse distances and $d\hat\sigma/dk_\perp^2$ of the small ones. At very high $k_\perp$ and s the hard cross-section or the S_0^2 can additionally contain the Sudakov like suppression factor depending on the scale corresponding to the border between small and large distances [4].

The cross-section of the hard $I\!\!P$-exchange $d\hat\sigma(s,t)/dt \sim g_1^2(t)g_2^2(t)$ is mainly determined by the behavior of $I\!\!P$-vertexes $g_j(t)$, because at large $-t$ the purely regge part

$$s^{\alpha(t)-1} \sim \exp(\ c\ \Delta_P\ \log s/\log(-t)\ )\ ,\quad c \sim 1 \tag{3}$$

grows very slowly with s. In perturbative QCD the hard part of $I\!\!P$ can be approximately represented as the BFKL ladder with mean interval between rapidities of emitted gluons of order of $\sim 1/\alpha_s(t)$. Such a BFKL type $I\!\!P$ exchange is almost of the same form as the direct 2n gluon exchange; the $n > 1$ contributions can arise because the BFKL pomeron contains also the multigluon t-channel contributions due to gluon reggeization [5]. And on this way we come finally to the quark counting type model.

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[4] In QCD the scale separating nonperturbative and the perturbative regions is probably located at rather high virtuality $\kappa \simeq 2 \div 3\ GeV$. In this case the perturbative Sudakov exponent contains factor $\alpha_s(\kappa)\log(k_\perp/\kappa)$, which is not sizeable.

[5] At high $-t$ not only $I\!\!P$ but also the other multigluon QCD reggeons can equally contribute to the hard exchange in the same way as BFKL pomeron $I\!\!P$ and odderon. Probably all such n-gluon trajectories $\alpha_n(t)$ at large $-t$ are approximately degenerated and accu-

The behavior of $g_i^2(t)$ at large $|t|$ in QCD can be estimated even quasi-classically, and this approach leads finally to the t dependence :

$$d\hat{\sigma}(s, k_\perp)/dk_\perp^2 \sim (\alpha_s(k_\perp))^\nu/(k_\perp^2)^N \quad , \tag{4}$$

with

$$\nu = n_1 + n_2 + |n_1 - n_2| \quad , \qquad N = \frac{1}{2}\big(3(n_1 + n_2) + |n_1 - n_2|\big) - 1 \quad ,$$

where n_1 and n_2 are numbers of valent constituents (fast quarks) in colliding hadrons. In (4) we have neglected the regge factor (3) , which at high $-t$ grows very slowly with s, and have also neglected the possible Sudakov type suppression factors - they can even approximately compensate one another. Expression (4) leads to the behavior of $\sim 1/t^8$ type for pp and to $\sim 1/|t|^7$ for πp cases.

There also can be another transverse configuration of constituents, in which the scattering index entering Eq.(4) is less then N. It corresponds to a scattering in a state in which the hadron constituents are arranged in the transverse plane on the line perpendicular to a scattering plane [6]. In such a case we have

$$N \;\rightarrow\; \hat{N} \;=\; \frac{5}{4}\,|n_1 + n_2| + \frac{3}{4}\,|n_1 - n_2| - \frac{1}{2} \quad , \tag{5}$$

which leads to the behavior $\sim 1/|t|^7$ type for pp and to $\sim 1/|t|^{13/2}$ for πp cases.

Note that the bare hadron cross-section $d\hat{\sigma}/dk_\perp^2$ given by (4) can be approximately interpreted as the cross-section of specific quasi-elastic process, where in the final state we have two hadrons with high transverse momenta $\simeq \pm k_\perp$, and all other particles have small $k_\perp^2 \sim \langle k_\perp^2 \rangle$. The particles

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mulate near intercept equal one. The difference in their contributions comes only from different values of the corresponding regge vertices.

[6]In fact all what is needed and what leads to expression (5) is the following : all valent constituents must scatter at closed angles $\theta \simeq k_\perp/k_z$ such that all relative transverse momenta of these partons be $\lesssim m$. Also one must fulfil the condition that after scattering all constituent partons should be located in the same packet of longitudinal size $\sim 1/k_z$. It leads to the condition that hadrons predominately scatter in specific initial configurations, when their partons are arranged on the lines in transverse planes. These lines must be perpendicular to the scattering plane; but the relative separation of partons on lines can be arbitrary. Such a picture of hard elastic scattering similar to the Landshoff mechanism [3]

which have large transverse momenta $k_\perp^2 \gg \langle k_\perp^2 \rangle$ and take more a than half of the total energy originate from hard interactions of leading partons. All other final particles are soft and come in the configuration typical for mean inelastic events at the same energy. These particles are created by the standard hadronisation of soft parton cloud, which follows after "removing" the leading partons in a hard interaction.

Such events also resemble the final state containing two high energy jets in special configurations, in which all the jets energy is concentrated on one fast particle (pion or nucleon). The probability of jet coming in such an "empty" state very likely contains the same damping factors $d\hat\sigma/dk_\perp^2$ and $S_0(s)$.

It can be interesting to single out similar events in high-energy heavy ion interactions. For example, in $A_1 A_2$ collisions those are the events with two nucleons of high transverse momenta $\pm k_\perp$ and soft rest hadrons with transverse momenta typical for soft $A_1 A_2$ collisions. The cross-sections of such a process is

$$d\sigma/dk_\perp^2 \simeq A_1 A_2 \cdot d\hat\sigma/dk_\perp^2 \ ,$$

because the nuclei are transparent for the bare nucleons.

3. The estimation of $|S_0|^2$ in parton models

Thus, to understand the behavior of $d\sigma/dt$ it is essential to estimate the behavior of $|S_0|$, which gives the probability that fast hadron is in a bare state, it is it contains only valent components and has has no additional parton cloud. For all accessible energies it is, mainly, the soft parton cloud connected with $I\!P$ exchange.

The $I\!P$ -ladder corresponds, roughly speaking, to a soft parton(gluon) cascading with some mean step in rapidity δ_y. Because these cascading steps are almost independent, the probability that no cascade be generated (it is a state without additional partons is realized) is of Poisson type $\sim \exp(-y\tilde\Delta)$, where $\tilde\Delta = c/\delta_y \ , \quad c \sim 1$. It corresponds to the fact that at y-boost the mean number of low energy partons $n(y)$ is defined by the linear equation $\partial n(y)/\partial y = \tilde\Delta n(y)$ [7]. So the crucial quantity is the value of $\tilde\Delta$, and we see

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[7]Evidently this is so for supercritical $I\!P$. If $\alpha_P(0)$ where < 1 then the probability find hadron in the "bare" state is always finite and dos not decrease with the hadron energy.

that $\tilde{\Delta} \simeq \delta_y^{-1}$ where δ_y is the mean step between essential degrees of freedom in the pomeron ladder. And one can estimate $|S_0(y)| \simeq \exp(-y/\delta_y)$.

These arguments can be presented in a slightly different way. To scatter at large $k_\perp$ hadrons should have the minimal parton clouds. It means that colliding particles must fluctuate to a state with small transverse size $r(y)$. The probability w of such a fluctuation is $\sim (r(y))^{2(\nu-1)}$, where ν is the number of valent constituents. In this case, the mean rapidity interval between steps in parton ladder would be $\delta_y \sim 1/\alpha_s(r(y))$. From the condition $\delta_y \simeq y$ (no partons except of valent ones in the whole rapidity interval) we find that $r(y) \sim \exp(-c_1 y)$, and we come to the same type of the exponential dependence $S_0(y) \sim w(y) \sim \exp(-2c_1(\nu - 1)y)$ as before [8].

Additional arguments for exponential behavior of probability $S_0(y)$ come from requirement of boost invariance of hard collision description in partonic terms. Let us consider this in arbitrary longitudinal frame, when the colliding particles have rapidities y_1 and y_2. Then the quantity $S_0(y)$, which is the amplitude that both colliding particles are in a bare state must have the multiplicative form

$$S_0(y) = S_0(y_1) \, S_0(y_2) \ ,$$

where $y = y_1 + y_2$. Because at boost $y_1 \to y_1 + \eta, \ y_1 \to y_1 - \eta, y \to y,$ the only functional form of $S_0(y)$ that fulfils this condition in arbitrary frame is exponential $S_0(y) \sim \exp(cy)$.

So to estimate the behavior of $S_0(y)$ we need to know the mean number of steps in the pomeron ladder. Two possibilities can be emphasized, and they correspond to a different choice of partons - the independent degrees of freedom in the Fock wave function of fast hadron.

In one case we can consider the Pomeron ladder as constructed from white particles (for example, from π and ρ mesons), as it is usually done in various multiperipheral type models. In this case the steps (parton) density in rapidity is $\simeq 0.5\rho \simeq 1$, where ρ is the mean density of produced hadrons at not too high energies, when the one $I\!\!P$ exchange dominates.

In other case one can identify the soft $I\!\!P$ ladder with the rare gluon ladder with steps $\tilde{\delta}_y \simeq \Delta_P^{-1}$, where $\Delta_P \simeq 0.1 \div 0.2$ is the "experimental" soft pomeron intercept [9].

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[8]This type of reasoning can remind the approach [6] to high energy scattering used in dual models embedded in the AdS_5 space.

[9]Such type of model was proposed [4] long ago, and in this case (with large δ_y) one can simply explain unnaturally small value of various pomeron parameters, such as

It seams, that the second possibility is theoretically more preferable, because the white hadron-partons do not represent the independent degrees of freedom, and therefore they are not completely appropriate to be used in the Fock space Hamiltonian.

If hard partons were also essential in the Fock w.f. then at first sight instead of $\simeq \exp(-y/\delta_y)$ one can expect a more complicated behavior $|S_0(y)|$. But the hard gluons are mainly taken into account in the parton w.f. as constituent of soft partons. It can be seen from the fact that at all acceptable energies the mean transverse momenta of secondary particles almost do not grow with energies. We come to the same conclusion from a different way by remarking that the hard gluon spectra "measured" in deep-inelastic reactions can be successfully described [5] as coming from renormalization group rescaling of the soft parton component.

At asymptotic energies the soft parton saturation can become fully essential and as a result the saturation scale $Q^{(sat)}(y)$ in transverse momenta can also become large. Then one should take into account in $|S_0|^2$ also more hard partons. Note that even in the in the asymptotical Froissart regime where saturation dominates the behavior of $|S_0|$ can be estimated in the same way as the probability that the valent components do not emit any primary gluons in the corresponding ranges of energy and transverse momenta $(q_\perp < \langle k_\perp \rangle \sim Q^{(sat)}(y))$. This is quite enough, because all other partons are emitted by these primaries. This evidently leads to a mean number of the primary partons

$$\bar{n}(y, k_\perp) \sim \int d\omega/\omega \int^{k_\perp} dq_\perp^2 \alpha_s(q_\perp)/q_\perp^2$$

and to Sudakov type factor for the no-emission probability $W(y, k_\perp) \sim \exp(-\bar{n})$, and, eventually, to the estimate $|S_0(y, k_\perp)| \simeq W(y_1, k_\perp)W(y_2, k_\perp)$, which is again boost-invariant.

4.   The estimation of $|S_0|^2$ in regge approach

The numerical values of $S_0 \equiv |S_0(y, b = 0)|$ can be estimated directly from the experimental data on behavior of the profile function $F(y, b) = 1 - S(y, b)$ at $b = 0$, calculated by the Fourier transformation of $\sqrt{(d\sigma/dt)^{(exper)}}$. For

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α_P', Δ_{Pm}, r_{3P},...

the pp and $p\bar{p}$ scattering it ranges from values $S_0 \simeq 0.6$ at $\sqrt{s} \sim 50 GeV$ up to values $S_0 \simeq 0.01 \div 0.02$ at $\sqrt{s} \sim 2\,TeV$.

In regge models the value of S_0 depends crucially on the relative weights of contributions of multipomeron exchanges to the elastic amplitude, and, in principle, can be extracted from "every" good model descriptions of $d\sigma/dt$ data at low t. But it is essential that in the most popular models one can not expect the functional behavior $S_0(y) \sim \exp(-yc)$ at large y, which seams very natural in the parton approach.

If $v(y,b)$ is the one $I\!P$ contribution to the amplitude, then the full sum of all pomeron diagrams (neglecting the pomeron interactions with one another) can be approximately represented as some function of v in the form

$$S(y,b) \;=\; S[v] \;=\; \sum_{n=0}^{\infty} \frac{\gamma_n}{n!}\,(iv)^n \;, \qquad \gamma_0 = \gamma_1 = 1 \;, \tag{6}$$

where the quantities $\gamma_n > 1$ for $n \geq 2$, and they take into account the contribution of diffractive jets in multipomeron vertices N_n.

In the simplest eikonal case, when all $\gamma_n = 1$ we have $S = \exp(iv)$ and it leads to $|S_0(y,b)| = \exp(-\texttt{Im}\ v(y,b)) \sim \exp(-c_1 \exp(\Delta y))$. Although such an eikonal-type amplitude (especially if properly adjusted [10]) can lead to reasonable description of many data, there will be the discrepancy with parton picture at very large y in any case.

The purely exponential behavior of $S(y,b)$ can take place only if the eikonal coefficients γ_n grow like $n!$ at large n. In this case we have for

$$|S_0(y,b)| \simeq (c + \texttt{Im}\ v(y,b))^{-1} \sim \exp(-\Delta y) \tag{7}$$

at large y [11].

The experimental data on the behavior of $d\sigma/dt$ at high $|t|$ are not rich, especially at high s. The old data on pp show [3] the universal behavior of $d\sigma/dt \simeq 0.1 t^{-8}$ for $2 \lesssim |t| \lesssim 15$ in the energy range $\sqrt{s} \simeq 30 \div 60 GeV$. At

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[10]The simplest "popular" form of corrections for the simple eikonal is the quasi-eikonal picture, in which one explicitly introduce the bare parton sate, or various multichannel eikonal models.

[11]For a reasonable behavior of series (6) the coefficients γ_n considered as function of n should be analytic for $Re\ n \geq 0$. The asymptotics of $S[v]$ for $v \to \infty$ is defined by most right singularities of γ_n over n. To have the asymptotics (7) the function γ_n must have a singularity at $n = -1$. The fairly interesting case corresponds to $\gamma_n = \Gamma(n+1)$ when $|S_0(y,b)| \simeq (1 + Im\ v(y,b))^{-1}$

such energies the S_0 is still rather large, and the decrease of $S_0(s)$ with s can in fact be compensated by the small growth (3) of $d\hat{\sigma}/dt$. New data at higher s and $-t$ are needed for better understanding the behavior of S_0.

5. Conclusion

We end with few remarks.

The quantity $|\tilde{S}(y \gg 1, b = 0)|$ which represents the probability that a fast hadron with energy $\simeq (m/2) \exp y$ has no soft parton cloud, is interesting from various aspects. This quantity can be extracted from the behavior of elastic cross section $d\sigma/dt$ at relatively low $|t|$. It can be also extracted from the data on the behavior of elastic cross-sections at relatively large $|t| > 2 \div 5 GeV^2$ (far outside the diffraction peak). All new data on elastic $d\sigma/dt$ are therefore very interesting from such a point of view, especially at maximally large (LHC) energies.

The existing data on $d\sigma/dt$ show that the value of $|S_0(y, b = 0)|$ is not so small, as one can expect at first sight from the large multiplicity of secondary hadrons at the same energies. It indicates that the high energy hadron wave function contains a relatively small number of partons in average. This also means that most of secondary soft hadrons are created only after collision (that can be a decay of the nonperturbative QCD tubes or minijets), and are not directly represented by the corresponding degrees of freedom in the incoming parton wave function. In most models of multiperipheral type the opposed picture is supposed usually

The comparison of the two different theoretical approaches to estimation of $|S_0(y, b = 0)|$ (based on parton picture and on multipomeron exchange model) suggests that the pure eikonal-type unitarization (when $S(y, b) = \exp(iv)$) is not suitable at very high energies. The most simple form, giving the same answer for $|S_0|$ as in the parton case, is given by the expression $S(y, b) = (1 + icv)^{-1}$, and it corresponds to the case when the eikonal coefficients grows ($\sim n!$) due to large contribution of diffractive jets in multipomeron vertices N_n. Note that such a grow of multi - $I\!\!P$ contributions, in comparison with the eikonal case, can essentially affect the form of the tail of multiplicity distribution [12] .

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[12] In the eikonal case the the behavior of the tail of multiplicity distribution is roughly $\sigma_n \sim \sigma_{inel}/\Gamma(1+n/\bar{n})$. In the case (7) the tail is larger: $\sigma_n \sim \sigma_{inel} \exp(-cn/\bar{n})$. Note that experimentally the tail has the form $\sigma_n \sim \exp(-3n/\bar{n})$. Here $\bar{n}$ is the mean multiplicity.

References

[1] S.J.Brodsky and G.R.Farrar Phys.Rev.Lett 31 (1973) 1153 ,

V.A.Matveev, R.M.Muradyan and A.N.Tavkhelidze, Lett.Nuovo Cim.7 (1973)719

[2] G.Sterman, arXiv:1008.4122 , M.G.Sotiropoulos and G.Sterman, hep-ph/9401237 , Nuclear Physics **B**419(1994)59 , **B**425(1994)489

[3] A. Donnachie and P.V. Landshoff, *Nucl.Phys.* **B244** (1984) 322; *Z. Phys.* **C 61** (1994) 139 , hep-ph/9607377

[4] V.A.Abramovsky and O.V.Kancheli, Pisma Zh.Eksp.Teor.Fiz.32:498-501,1980.

[5] A.Capella, E.G.Ferreiro, C.A.Salgado, A.B.Kaidalov, hep-ph/0005049, hep-ph/0006233

[6] J.Polchinski and M.J.Strassler, arXiv:hep-th/0209211 ,

R.C.Brower, J.Polchinski, M.J.Strassler, C.I.Tan, JHEP**0712**, 005, 2007 [arXiv:hep-th/0603115]